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Cosmological Background in Higgs Scalar–Tensor Theory Without Higgs Particles

H. Frommert[1,2] and H. Schoor[1]

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The scalar background field and its consequences are discussed for the Friedmann-type cosmological solutions of the scalar–tensor theory of gravity with the Higgs field of the Standard Model as the scalar gravitational field.

1. INTRODUCTION

In their scalar–tensor theory of gravity, Brans and Dicke [3], as well as successors including Bergmann [2] and Wagoner [10], replaced the Newtonian gravitational constant by a scalar field, i.e., function, which, within their model, enters the theory as a completely new field.

However, modern physics already deals with another scalar field, the Higgs field of the Standard Model of particle physics. As was first proposed by Zee [11], it appears appealing to use this scalar Higgs field of particle physics also as the scalar field in a scalar–tensor theory of gravity; this approach has been investigated by Dehnen *et al.* [6]. In this theory, in addition to its role in the Standard Model to make the particles massive, the scalar Higgs field also generates the gravitational constant G, in the sense discussed in Adler's review article [1] of generating an 'induced' G from symmetry breaking.

Surprisingly, however, if the Higgs field of the $SU(3) \times SU(2) \times U(1)$ Standard Model of the elementary particles is employed to generate G, the Higgs field loses its source, i.e., can no longer be generated by fermions and gauge bosons unless in the very weak gravitational channel [7]. Similar results

[1] Department of Physics, University of Constance, D-78434 Konstanz, Germany; e-Mail: spider@seds.org.
[2] H. Frommert is now at DMC GmbH, Wamslerstr. 5, D-81829 Mürchen, Germany.

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were obtained independently by van der Bij [8]. As Styp-Rekowski and Frommert [9] have shown, the only physically meaningful *static* solution of this theory is the trivial one without any excited Higgs field present. As the physical world is not static, a potential scalar field of the cosmological background is most interesting, both on its own (e.g., for inflational scenarios; see Cervantes-Cota and Dehnen [4]) and in order to have a fit for a more realistic physical (i.e., dynamical) model configuration (e.g., galaxy).

Here we investigate the cosmological background scalar field for Friedmann-type cosmologies analytically. After a general discussion of the cosmological field equations, we implicitly assume weak scalar fields and discuss the solutions of the scalar field equation for given expansion laws. Finally, we obtain effective Einstein field equations as well as effective, time-dependent values for the gravitational and the cosmological "constant," as well as an effective vacuum energy density.

The reader can find the whole underlying formalism of this theory in Dehnen and Frommert [7].

2. FRIEDMANN-TYPE COSMOLOGY IN THE HIGGS SCALAR–TENSOR THEORY

As outlined in ref. 7, the Higgs scalar–tensor theory can be obtained by adding the scalar–tensor gravitational Lagrange density to the matter Lagrangian of particle physics, which must be taken in curved spacetime here, of course. After performing the symmetry breaking, one arrives at the following Lagrange density[3]:

$$\mathcal{L} = \left\{\frac{\alpha v^2}{16\pi}\left[(1+\varphi)^2 R - 2\Lambda\right] + \frac{v^2}{2}\varphi_{|\mu}\varphi^{|\mu} - V(\varphi) + L_M\right\}(-g)^{1/2} \quad (1)$$

where φ is the excited Higgs field, R the Ricci scalar corresponding to, and g the determinant of the metric $g_{\mu\nu}$, $V(\varphi)$ the Higgs potential, and L_M the effective matter Lagrangian after symmetry breaking of the fermions and gauge bosons of the Standard Model of particle physics (see, e.g., ref. 5). v is the constant absolute (real) value of the vacuum scalar field, or Higgs field ground state, and α is a numerical constant which, for the standard model Higgs field considered here, is given by the square of the ratio of the Planck mass to the mass of the electroweak W bosons:

[3] Throughout this paper we use $\hbar = c = 1$ and the metric signature $(+ - - -)$. The symbol $(\ldots)_{|\mu}$ denotes the partial, $(\ldots)_{\|\mu}$ the covariant derivative with respect to the coordinate x^{μ}. For the cosmological discussion here, we also include the cosmological constant Λ, which was omitted in previous works.

$$\alpha \simeq (M_{Pl}/M_W)^2 \simeq 10^{33} \tag{2}$$

With the convenient substitution for the excited Higgs field $\xi = (1 + \varphi)^2 - 1$, the Higgs potential $V(\xi)$ is defined by

$$V(\xi) = \frac{3}{32\pi G} M^2 \left(1 + \frac{4\pi}{3\alpha}\right) \xi^2 \approx \frac{3M^2}{32\pi G} \xi^2 \tag{3}$$

From the variation of the Lagrange density (1), one obtains for the excited Higgs field ξ the following homogeneous, covariant Klein–Gordon equation [7]:

$$\xi_{\|\mu}^{|\mu} + M^2\xi - \frac{\frac{4}{3}\Lambda}{1 + 4\pi/3\alpha} = 0 \tag{4}$$

where M denotes the mass of the Higgs particles in this theory.[4] The field equation for the metric as the tensorial gravitational field reads

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \frac{\Lambda}{1 + \xi} g_{\mu\nu} \\ = -\frac{8\pi G}{1 + \xi} \left[T_{\mu\nu} + \frac{v^2}{4(1 + \xi)} \left(\xi_{|\mu}\xi_{|\nu} - \frac{1}{2} \xi_{|\lambda}\xi^{|\lambda} g_{\mu\nu} \right) + V(\xi) g_{\mu\nu} \right] \\ - \frac{1}{1 + \xi} [\xi_{|\mu\|\nu} - \xi_{\|\lambda}^{|\lambda} g_{\mu\nu}] \end{aligned} \tag{5}$$

with the Ricci tensor $R_{\mu\nu}$ belonging to the metric $g_{\mu\nu}$ and $T_{\mu\nu}$ as energy-momentum tensor of matter. Because of the very large value of α (which is responsible for the relative weakness of gravity as well as for the rather small Higgs mass in this theory), we will neglect the $1/\alpha$ terms in equations (4) and (3) in the following.

Here we look for solutions of the scalar field equation (4) on the cosmological background of a Robertson–Walker metric, defined by the line element (in isotropic spatial coordinates)

$$ds^2 = dt^2 - a(t)^2 \frac{1}{1 - (\varepsilon/4)r^2} (dr^2 + r^2 d\Omega^2) \tag{6}$$

where, as usual, $\varepsilon = 0, +1, -1$ corresponds to the spatially flat, the closed, and the open model universe, respectively, and $a(t)$ is the time-dependent scale factor. Moreover, we approximate the matter in cosmos by a perfect fluid, characterized by its density ρ and pressure p only, as usual, and demand

[4] This equation is homogeneous, as the constant-Λ term may be absorbed in the excited Higgs field ξ.

that ρ and p as well as the scalar field ξ are functions of the time coordinate t only (cosmological principle). Then the scalar field equation (4) becomes[5]

$$\ddot{\xi} + 3\frac{\dot{a}}{a}\dot{\xi} + M^2\xi - \frac{4}{3}\Lambda \equiv 0 \tag{7}$$

while the nontrivial components of the Einstein equations are the Friedmann equations:

$$\frac{\dot{a}^2 + \varepsilon}{a^2} - \frac{\Lambda/3}{1+\xi} = \frac{8\pi G}{3}\frac{1}{1+\xi}\left[\rho + \frac{v^2/8}{1+\xi}\dot{\xi}^2\right] + \frac{1}{1+\xi}\left[-\frac{\dot{a}}{a}\dot{\xi} + \frac{1}{4}M^2\left(1 + \frac{4\pi}{3\alpha}\right)\xi^2\right] \tag{8}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + \varepsilon}{a^2} - \frac{\Lambda}{1+\xi} = -\frac{8\pi G}{1+\xi}\left[p + \frac{v^2/8}{1+\xi}\dot{\xi}^2\right] - \frac{1}{1+\xi}\left[\ddot{\xi} + 2\frac{\dot{a}}{a}\dot{\xi} - \frac{3}{4}\left(\frac{M}{\hbar}\right)^2\left(1 + \frac{4\pi}{3\alpha}\right)\xi^2\right] \tag{9}$$

These equations are augmented by the equation of continuity for the energy-momentum tensor, which here reduces to

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = \frac{\dot{\xi}}{2(1+\xi)}(\rho - 3p) \tag{10}$$

For the discussion of the cosmological background scalar field, one notes first that, up to terms proportional to the Hubble constant $H = \dot{a}/a$ and the cosmological constant Λ, the scalar field equation (7) is solved by the periodic function

$$\xi = \xi_0 \cos[\omega(t - t_0)], \qquad \omega = \omega_M = M \quad \left(= \frac{Mc^2}{\hbar}\right) \tag{11}$$

with a constant amplitude ξ_0 and period ω_M, which is the Compton frequency belonging to the Higgs mass M, and thus *very* large compared to the Hubble constant. One could expect modifications of ξ_0 and ω_M for the exact solutions which are time dependent, but change significantly only on the cosmological time scale, so that for all noncosmological considerations equation (11) should be a good approximation.

For the following, more detailed discussion, we note that the term with the cosmological constant Λ in the scalar field equation (7) is even small

[5] $(\ldots)^{\cdot} := (\partial/\partial t)(\ldots)$ denotes the time derivative of $(\ldots)$.

compared to the one containing the Hubble constant, and can be absorbed in ξ as a small additive contribution. Therefore, we can restrict ourselves here to discuss the equation for $\Lambda = 0$ only, which reads

$$\ddot{\xi} + 3\frac{\dot{a}}{a}\dot{\xi} + M^2\xi = 0 \tag{12}$$

This equation may be simplified with the ansatz

$$\xi(t) =: a^{-3/2}u(t) \tag{13}$$

Herewith the field equation (12) reads

$$\ddot{u} + \left[M^2 - \frac{3}{4}\left(\frac{\dot{a}}{a}\right)^2 - \frac{3\ddot{a}}{2a}\right]u = 0 \tag{14}$$

Up to terms of order H^2 and $\ddot{a}/a = -qH^2$ (with the cosmological acceleration parameter[6] q), the solution of this equation is identical to equation (11). Using the cosmological parameters, the Hubble constant H and the acceleration parameter q, we obtain equation (14) in the form

$$\ddot{u} + \left[M^2 + \frac{3}{2}(2q - 1)H^2\right]u = 0 \tag{15}$$

For times small compared to the Hubble time $1/H$, this equation should be approximately solved by the solutions (11) of the approximated equation (7), so that

$$u = u_0 \cos[M(t - t_0)], \qquad \xi = u_0 a^{-3/2} \cos[M(t - t_0)] \tag{16}$$

This solution is accurate to one order more (i.e., to the second order) in H, or considered time differences compared to the Hubble time, than solution (11). Some higher accuracy can be obtained by inserting the current values of the function-valued parameters H and q into equation (15):

$$\ddot{u} + \left[M^2 + \frac{3}{2}(2q_0 - 1)\,H_0^2\right]u = 0, \tag{17}$$
$$q_0 = q(t_0) = \text{const}, \qquad H_0 = H(t_0) = \text{const}$$

which is solved by

[6] q is half the density parameter Ω in standard Friedmann cosmology without cosmological Λ.

$$u = u_0 \cos\left[\sqrt{1 + \frac{3}{2}(2q_0 - 1)\left(\frac{H_0}{M}\right)^2} M(t - t_0)\right]$$

$$\xi = u_0 a^{-3/2} \cos\left[\sqrt{1 + \frac{3}{2}(2q_0 - 1)\left(\frac{H_0}{M}\right)^2} M(t - t_0)\right] \qquad (18)$$

This solution differs from the above equation (16), by a slightly different oscillation frequency, deviating from that in equation (16) (which is the Compton frequency corresponding to the Higgs mass M) by a correction of the order given by the square of the ratio[7] of the two characteristic times relevant here, the Compton time $1/M$ corresponding to the Higgs mass M, and the Hubble time $1/H_0$: $(H_0/M)^2$. The smallness of this value can already be seen by estimating the corrective term in the field equation (17), or the frequency in (18), using the second Friedmann equation (9), which yields, neglecting the scalar field ξ,

$$(2q - 1)H^2 = \frac{\varepsilon}{a^2} + 8\pi G p + \Lambda$$

i.e., it is determined by the largest of its three terms: If $\varepsilon = \pm 1$ (nonflat case), baryonic matter dominates ($p \ll \rho$), and $\Lambda \ll a^{-2}$, it is essentially given by $\pm 1/a^2$, while for the flat case, either the weak matter pressure p or the cosmological constant Λ determines this correction. In view of this small value, one has to consider that for a self-consistent solution, the contributions of the scalar field in the Friedmann equations (8) and (9) must be taken into account, and it is not guaranteed that these are smaller than the deviations discussed here. This will be important for attempts to iterate cosmological solutions in this theory.

One may also discuss the *exact* solutions. A self-consistent, exact, and simultaneous solution of the relevant equations (8)–(10) and (12) cannot be given analytically. However, it can be obtained numerically to some approximation, which was done by Cervantes-Cota and Dehnen [4], and is of particular interest, especially in the context of possible inflation scenarios. Here we discuss analytic background solutions, which are obtained if a Friedmann solution is given as external field. Then it is possible to solve equation (16) analytically for each given ansatz for $a(t)$. This is presented in the following for the two simplest cases, where it is possible to give

[7] The value of this ratio can be estimated, taking into account that the Higgs mass in this theory is smaller than the usual one by a factor of about 2.5×10^{16}, or at least about 2.5×10^{-6} eV/c^2, corresponding to a Compton time of $\hbar/Mc^2 \approx 2.6 \times 10^{-10}$ s. This must be compared to the Hubble time of 13 billion years [assuming $H_0 = 75$ *km*/(s *Mpc*)], so that $(H_0/M)^2 \approx 4 \times 10^{-55}$.

the exact solutions (also mentioned in ref. 4 as the limiting cases for the inflationary cosmology):

1. The spatially flat Friedmann universe with $\Lambda = 0$,
2. The empty universe with cosmological constant Λ.

One may hope that these background solutions are of interest in considerations where cosmology plays the role of a background, i.e., the dynamics of galaxies or clusters of galaxies.

2.1. The Spatially Flat Friedmann Universe with $\Lambda = 0$

In this case, we have the following time evolution of the scale parameter $a(t)$:

$$a(t) = At^{2/3} \tag{19}$$

and therefore

$$H(t) = \frac{\dot{a}}{a} = \frac{2}{3t} \tag{20}$$

$$\frac{\ddot{a}}{a} = -\frac{2}{9t^2} \qquad \left(q = \frac{1}{2}\right) \tag{21}$$

This ansatz leads to the following equation for ξ:

$$\xi = a^{-3/2}u = A^{-3/2}t^{-1}u$$

The differential equation (14) for u simplifies exactly to

$$\ddot{u} + M^2 u = 0 \tag{22}$$

which is identical to the equation without H_0, and has the exact solution

$$u(t) = u_0 \cos[M(t - t_0)] \tag{23}$$

or

$$\xi = A^{-3/2}u_0 t^{-1} \cos[M(t - t_0)] = \xi_0 t_0 \frac{\cos[M(t-t_0)]}{t} \tag{24}$$

where the relation for the amplitude at present time, $\xi_0 = A^{-3/2}u_0/t_0 = a_0^{-3/2}u_0$, was used. To see the approximate constancy of the amplitude over times small compared to the Hubble time (or world age), one may expand the time t as $t = t_0 + T$. Then the solution (24) can be rewritten as

$$\xi = \xi_0(t)\cos[M(t - t_0)], \qquad \text{where} \quad \xi_0(t) = \frac{\xi_0}{1 + T/t_0} \approx \xi_0\left(1 - \frac{3}{2}H_0T\right) \tag{25}$$

2.2. The Empty Universe with Nonvanishing Λ (de Sitter Universe)

As a second ansatz, we investigate the case of the matter-free universe with nonvanishing Λ (matter influence negligible compared with that of the cosmological constant). Then we have

$$H = \frac{\dot{a}}{a} = \sqrt{\Lambda/3} = \text{const}, \qquad a = a_0 e^{H(t-t_0)}, \qquad q = -1 \tag{26}$$

With this ansatz, the differential equation (14) for u takes the form

$$\ddot{u} + \left(M^2 - \frac{9}{4}H^2\right)u = 0 \tag{27}$$

which has the solution

$$u = u_0 \cos\left[\sqrt{M^2 - \frac{9}{4}H^2}(t - t_0)\right] \tag{28}$$

or

$$\xi = a^{-3/2}u = \xi_0 e^{-(3/2)H(t-t_0)} \cos\left[\sqrt{M^2 - \frac{9}{4}H^2}(t - t_0)\right] \tag{29}$$

Again, with $t = t_0 + T$, the time-dependent amplitude $\xi_0(t)$ of this solution [see (25)] can be expanded in orders of T/t_0 or Ht, and has the same form as above up to the first order: $\xi_0(t) = \xi_0(1 - \frac{3}{2}HT)$.

3. REMARKS ON THE EXACT SOLUTIONS

The scalar field equation (14) or (15), in view of the approximate solutions (16) or (18), may be rewritten with the ansatz $u = u_0(t)\cos[\beta(t)]$, where $\beta(t) = M \int^t dt' \sqrt{1 + \kappa(t')}$, so that $\dot{\beta} = M\sqrt{1 + \kappa(t)}$ ($\kappa \ll 1$ if $H/M \ll 1$, $t_H \gg \hbar/Mc^2$). Then one obtains

$$\ddot{u}_0 + \left[-M^2\kappa + \frac{3}{4}(2q - 1)H^2\right]u_0 = 0 \tag{30}$$

$$\dot{\beta}u_0^2 = M\sqrt{1 + \kappa u_0^2} = \text{const} =: C \tag{31}$$

The second of these equations, (31), is solved, e.g., by $u_0 = const$ and $\dot{\beta} = const$ or $\kappa(t) = const$; this is possible if q and H in (30) are taken as constants. According to the remarks above, the deviation of u_0 and $\dot{\beta}$ or $\kappa(t)$ from constancy can be expected to be small, at least for the large Hubble times considered here. One may rewrite (30) by evaluating (31) in a single nonlinear differential equation for $u_0(t)$:

$$\ddot{u}_0 + \left[M^2 + \frac{3}{4}(2q - 1)H^2 \right] u_0 - \frac{C^2}{u_0^3} = 0 \tag{32}$$

In view of the very small values of the corrections to the approximate solution (16) or (18), we will not further discuss this equation here.

4. EFFECTIVE EINSTEIN EQUATIONS WITH AN AVERAGED COSMOLOGICAL SCALAR FIELD

Having on hand that the scalar field solution for the cosmological background is given by a rapidly oscillating function which overlies an amplitude which changes at cosmological time scales only, it is of interest to take a new look at the basic field equations of the theory. With the scalar field amplitude given by

$$\xi_0 = a(t)^{-3/2}\, u_0 = \xi_0(t) \tag{33}$$

where $u_0 = u_0(t)$ must fulfill the differential equation (30) or (32), the time-averaged[8] (over one period of the scalar oscillation) Einstein equations take the form ("effective" Einstein equations):

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \left\langle \frac{\Lambda}{1+\xi} \right\rangle g_{\mu\nu} &\\ = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \frac{\Lambda}{\sqrt{1 - \xi_0(t)^2}}\, g_{\mu\nu} &\\ = \left\langle -\frac{8\pi G}{1+\xi} \left[T_{\mu\nu} + \frac{v^2}{4(1+\xi)} \left(\xi_{,\mu}\xi_{,\nu} - \frac{1}{2}\xi_{,\lambda}\xi^{,\lambda} g_{\mu\nu} \right) + V(\xi) g_{\mu\nu} \right] \right. & \end{aligned}$$

[8] The averaged terms are obtained by Taylor-expanding the factors corresponding to the 1 + ξ denominators so that power series of ξ are obtained, and then taking the time average given by the formula

$$\langle F(t) \rangle = \langle F \rangle (t_0) = \frac{1}{2\pi} \int_{t_0}^{t_0+T} F(t')\, dt'$$

where $T = 1/\omega$ is the oscillation period, which must be small compared to the age of the universe t_0. This average is a time function which varies slowly over cosmological time scales.

$$- \frac{1}{1+\xi}\left(\xi_{,\mu;\nu} - \frac{1}{2}\xi_{,\lambda}^{;\lambda} g_{\mu\nu}\right)\Bigg\rangle$$

$$\stackrel{!}{=} -\frac{8\pi G}{\sqrt{1-\xi_0(t)^2}} T_{\mu\nu} - \delta_\mu^0\delta_\nu^0 M^2\left(\frac{1}{\sqrt{1-\xi_0(t)^2}} - 1\right)\left(1 - \frac{2\pi}{\alpha}\right)$$

$$+ \frac{M^2}{4} g_{\mu\nu}\left(\frac{1}{\sqrt{1-\xi_0(t)^2}} - 1\right)$$

This means that the scalar field leads to the following:

- A (time-dependent) effective gravitational "constant"

$$G_{\text{eff}} = \frac{G}{\sqrt{1-\xi_0(t)^2}} > G \tag{35}$$

- A correction factor and a negative (attractive) contribution to the cosmological constant, or function, which becomes effectively

$$\Lambda_{\text{eff}} = \frac{\Lambda}{\sqrt{1-\xi_0(t)^2}} - \frac{M^2}{4}\left(\frac{1}{\sqrt{1-\xi_0(t)^2}} - 1\right) \tag{36}$$

- A positive effective "energy (or mass) density of the vacuum," given by

$$T_{\mu\nu}^{\text{vac}} = \delta_\mu^0\delta_\nu^0 \rho_{\text{vac}} = \frac{\delta_\mu^0\delta_\nu^0 M^2}{8\pi G_{\text{eff}}}\left(\frac{1}{\sqrt{1-\xi_0(t)^2}} - 1\right)\left(1 - \frac{2\pi}{\alpha}\right) \tag{37}$$

As $\xi_0(t)$ changes over cosmological time scales only, it may be regarded as constant in a first approximation unless cosmological aspects are discussed for themselves. This may be of particular interest for the dynamics of galaxies (e.g., rotation curves) and galaxy clusters, i.e., the dark matter problem. The calculation of the rotation curves for some disk galaxy models is in preparation.

A possible limit for the time variations of $\xi_0(t)$ may be found from geophysical or solar system results.

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